

Hom(Hom(-, Z128), Z16)_On_Seq

Here we demonstrate on a simple example some main procedures in `homalg`. We start with the principal ideal ring $D := \mathbb{Z}/2^8\mathbb{Z}$ and the short exact sequence of modules

$$(0 \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow 0) = (0 \rightarrow \mathbb{Z}/2^2\mathbb{Z} \rightarrow \mathbb{Z}/2^5\mathbb{Z} \rightarrow \mathbb{Z}/2^3\mathbb{Z} \rightarrow 0).$$

The functor we want to derive is the composed functor

$$F := \text{Hom}(\text{Hom}(-, K), L) = \text{Hom}(\text{Hom}(-, \mathbb{Z}/2^7\mathbb{Z}), \mathbb{Z}/2^4\mathbb{Z})$$

. We compute the long exact sequence of the left derived functors $L_i F$:

```

> restart;
> with(PIR): with(homalg):
> RPP:='PIR/homalg': 'homalg/default':=RPP;
      homalg/default := PIR/homalg
> var:=[[ ], [2^8]]: Pvar(var);
      ["Z", 256]
> M:=Cokernel([2^5],var); alpha2:=matrix([[1]]);
_M:=Cokernel([2^3],var);
      M := [[1 = 1], [32], "Presentation", [32], 0]
      alpha2 := [ 1 ]
      _M := [[1 = 1], [8], "Presentation", [8], 0]
> M_:=Kernel(M,alpha2,_M,var); alpha1:=KernelEmb(M,alpha2,_M,var);
      M_ := [[1 = 8], [4], "Presentation", [4], 0]
      alpha1 := [ 8 ]
> IsShortExactSeq(M_,alpha1,M,alpha2,_M,var);
      true
> K:=Cokernel([2^7],var); L:=Cokernel([2^4],var);
      K := [[1 = 1], [128], "Presentation", [128], 0]
      L := [[1 = 1], [16], "Presentation", [16], 0]
> GlobalDim(var); q:=4;
      infinity
      q := 4
> seqs:=ResolveShortExactSeq(q,M_,alpha1,M,alpha2,_M,var,"TRUNCATE"):
> Seqs:=HomHomOnSeqs(K,L,seqs,var):
> LEHS:=LongExactHomologySeq(Seqs,var):
> map(a->LHomHomMap(a,M_,alpha1,M,K,L,var),[$0..q]);
map(a->LHomHomMap(a,M,alpha2,_M,K,L,var),[$0..q]);
      [[ 8 ], [ 4 ], [ 4 ], [ 4 ], [ 4 ]]
      [[ 1 ], [ 2 ], [ 2 ], [ 2 ], [ 2 ]]

```

We obtain the long exact sequence of derived functors:

$$0 \leftarrow \mathbb{Z}/8\mathbb{Z} \xleftarrow{(7)} \mathbb{Z}/16\mathbb{Z} \xleftarrow{(8)} \mathbb{Z}/4\mathbb{Z} \xleftarrow{(2)} \mathbb{Z}/8\mathbb{Z} \xleftarrow{(6)} \mathbb{Z}/8\mathbb{Z} \xleftarrow{(4)} \mathbb{Z}/4\mathbb{Z} \xleftarrow{(2)} \mathbb{Z}/8\mathbb{Z} \xleftarrow{(6)} \mathbb{Z}/8\mathbb{Z} \xleftarrow{(4)} \dots \text{periodic}$$

```

> lehs:=LEHS2lehs(LEHS);

```

```

lehs := [[[1 = [ 1 ]], [8], "Presentation", [8], 0], [ 7 ] ,
[[1 = [
  7
 249 ]], [16], "Presentation", [16], 0], [ 8 ] ,
[[1 = [ 1 ]], [4], "Presentation", [4], 0], [ 2 ] , [[1 = [ 2 ]], [8], "Presentation", [8], 0],
[ 6 ] , [[1 = [
  0
  1 ]], [8], "Presentation", [8], 0], [ 4 ] ,
[[1 = [ 4 ]], [4], "Presentation", [4], 0], [ 2 ] , [[1 = [ 1 ]], [8], "Presentation", [8], 0],
[ 6 ] , [[1 = [
  6
 250 ]], [8], "Presentation", [8], 0], [ 4 ] ,
[[1 = [ 1 ]], [4], "Presentation", [4], 0], [ 2 ] , [[1 = [ 2 ]], [8], "Presentation", [8], 0],
[ 6 ] , [[1 = [
  0
  1 ]], [8], "Presentation", [8], 0], [ 4 ] ,
[[1 = [ 4 ]], [4], "Presentation", [4], 0], [ 2 ] , [[1 = [ 1 ]], [16], "Presentation", [16], 0],
[
 14
  0 ] , [[[1, 0] = [
 254
  2 ] , [0, 1] = [
  0
 15 ]], [[8, 0], [0, 16]], "Presentation", [8, 16], 0], [ 0 1 ] ,
[[1 = [ 1 ]], [16], "Presentation", [16], 0]]
> IsExactSeq(lehs,var,"VERBOSE");
true

```

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REFERENCES

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- [BR] Mohamed Barakat and Daniel Robertz, *homalg - A meta-package for homological algebra*, submitted. [arXiv:math.AC/0701146](https://arxiv.org/abs/math/0701146) and (<http://wwwb.math.rwth-aachen.de/homalg>).
- [BR07] ———, *homalg project*, 2004-2007, (<http://wwwb.math.rwth-aachen.de/homalg>).

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