ExtensionWithoutFilteredModule

Theorem 5.1 of [BB] states that a 2-extension module exists iff the YONEDAproduct of the given 1-cocycles vanishes. In this worksheet we will give examples that this condition is in fact necessary.

```
>  restart;
with(Involutive):  with(homalg):
'homalg/default':='Involutive/homalg';
```

$$homalg/default := Involutive/homalg$$

Let $D = \mathbb{Q}[x, y]$.

```
>  var:=[x,y];
```

$$var := [x, y]$$

We choose M, L and N as follows:

```
>  M := [x,y];
```

$$M := [x, y]$$

```
>  L := [[x,y]];
```

$$L := [[x, y]]$$

```
>  N := [x];
```

$$N := [x]$$

```
>  Ext(1,M,L,var);
```

$$[[1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}], [y, x], \text{``Presentation''}, 1, [0, 0]]$$

```
>  Ext(1,L,N,var);
```

$$[[1 = \begin{bmatrix} 1 \end{bmatrix}], [y, x], \text{``Presentation''}, 1, [0, 0]]$$

The two 1-cocycles are

```
>  etaML := matrix([[1, 0], [0, -1]]);
```

$$etaML := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

```
>  etaLN := [1];
```

$$etaLN := [1]$$

The YONEDAproduct of these cocycles does not vanish

```
>  YonedaProductOfCocycles(M,etaML,1,L,etaLN,1,N,var,"return_abstract_ge
nerator");
```

$$1 = \begin{bmatrix} 1 \end{bmatrix}$$

so no 2-extension module exists and $\text{ExtMod}(\eta_L^M, \eta_N^L) = \emptyset$:

```
>  A2ExtensionModule(M,etaML,L,etaLN,N,var);
```

$$FAIL$$

Another example:

```
>  M := [x^3, y^2+1];
```

$$M := [x^3, y^2 + 1]$$

```
>  N := [[x,y*x]];
```

$$N := [[x, y\,x]]$$

```
>  Ext(2,M,N,var);
```

$$[[[1, 0] = \begin{bmatrix} 0 & 1 \end{bmatrix}, [0, 1] = \begin{bmatrix} 1 & 0 \end{bmatrix}],$$
$$[[0, y^2 + 1], [y^2 + 1, 0], [-x, y\,x], [y\,x, x], [-x^2, x^2\,y], [x^2\,y, x^2], [0, x^3], [x^3, 0]],$$
$$\text{``Presentation''}, 2 + 4\,s + 2\,s^2, [0, 0]]$$

We take a 2-cocycle that is not trivial in $\mathrm{Ext}^1(M, N)$ and compute the corresponding 2-extension:

```
>  eta := [[1,0]];
```

$$\eta := [[1, 0]]$$

```
>  ext2 := Extension(2,M,eta,N,var);
```

$ext2 := [[[1, 0] = [1, 0], [0, 1] = [0, 1]], [[x,\, y\,x]], \text{"Presentation"},$

$2 + 4\,s + s^2\,(\dfrac{4}{1-s} + \dfrac{1}{(1-s)^2}),\, [4,\, 1]],\, \begin{bmatrix} x^3 & -1 - y^2 & 0 \\ 0 & 0 & -1 \end{bmatrix},\, [$

$[[1, 0, 0] = [1, 0, 0, 0], [0, 1, 0] = [0, 1, 0, 0], [0, 0, 1] = [0, 0, 0, 1]],$

$[[x^4,\, -x - y^2\,x,\, -y\,x]],\, \text{"Presentation"}\ ,\, 3 + 6\,s + 9\,s^2 + 12\,s^3 + s^4\,(\dfrac{12}{1-s} + \dfrac{2}{(1-s)^2}),$

$[12, 2]],\, \begin{bmatrix} y^2 + 1 \\ x^3 \\ 0 \end{bmatrix},\, [[1 = 1], [0], \text{"Presentation"},\, \dfrac{1}{(1-s)^2},\, [0, 1]],\, \begin{bmatrix} 1 \end{bmatrix},$

$[[1 = 1],\, [y^2 + 1,\, x + y^2\,x,\, x^3,\, x^2 + y^2\,x^2],\, \text{"Presentation"},\, 1 + 2\,s + 2\,s^2 + s^3,\, [0, 0]]$

And indeed, there is no 2-extension module inducing `ext2`:

```
>  A2ExtensionModuleFromA2Extension(ext2,var);
```

$$FAIL$$

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REFERENCES

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[BR] Mohamed Barakat and Daniel Robertz, `homalg` – *A meta-package for homological algebra*, accepted for publication in Journal of Algebra and its Applications. (arXiv:math.AC/0701146 and http://wwwb.math.rwth-aachen.de/homalg).

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