BZ

```
> restart:
> with(Involutive): with(homalg):
```

Tell `Involutive` to compute over the integers:

```
> InvolutiveOptions("rational",false);
```

$$true$$

```
> RPI:=`Involutive/homalg`;
```

$$RPI := Involutive/homalg$$

```
> `homalg/default`:=RPI;
```

$$homalg/default := Involutive/homalg$$

The infinite cyclic group $\mathbb{Z} := (\mathbb{Z}, +) = C_\infty$:
The group ring $\mathbb{Z}\mathbb{Z} = \mathbb{Z}C_\infty = \mathbb{Z}[x, x^{-1}]$:

```
> var:=[[x,X],[x*X-1]];
```

$$var := [[x,\ X],\ [x\,X - 1]]$$

The trivial $\mathbb{Z}C_\infty$-module $\mathbb{Z}$:

```
> Z:=[x-1];
```

$$Z := [x - 1]$$

THEOREM: *For a discrete group G there are natural isomorphisms*

$$\mathrm{Ext}^\bullet_{\mathbb{Z}G}(\mathbb{Z},\mathbb{Z}) = \mathrm{H}^\bullet(G,\mathbb{Z}) = \mathrm{H}^\bullet(\mathrm{B}G;\mathbb{Z}).$$

The integral cohomology of the group $\mathbb{Z}^n$ = integral cohomology of the classifing space

$$\mathrm{B}Z = (S^1)^n = T^n,$$

the n-dimensional torus.
THEOREM: *The cohomology ring of $\mathbb{Z}^n$ is* $\mathrm{Ext}^\bullet_{\mathbb{Z}\mathbb{Z}^n}(\mathbb{Z},\mathbb{Z}) = \mathrm{H}^\bullet(\mathrm{B}\mathbb{Z}^n;\mathbb{Z}) = \mathrm{H}^\bullet(T^n;\mathbb{Z}) = \Lambda\mathbb{Z}^n$.
The zeroth integral cohomology $\mathrm{H}^0(S^1;\mathbb{Z}) = \mathbb{Z}$:

```
> Ext(0,Z,Z,var);
```

$$[[1 = \begin{bmatrix} 1 \end{bmatrix}],\ [X - 1,\ x - 1],\ \text{“Presentation”}]$$

The first integral cohomology $\mathrm{H}^1(S^1;\mathbb{Z}) = \mathbb{Z}$:

```
> Ext(1,Z,Z,var);
```

$$[[1 = \begin{bmatrix} 1 \end{bmatrix}],\ [X - 1,\ x - 1],\ \text{“Presentation”}]$$

The second integral cohomology $\mathrm{H}^2(S^1;\mathbb{Z}) = 0$:

```
> Ext(2,Z,Z,var);
```

$$[[1 = \begin{bmatrix} 0 \end{bmatrix}],\ [1],\ \text{“Presentation”}]$$

The third integral cohomology $\mathrm{H}^3(S^1;\mathbb{Z}) = 0$:

```
> Ext(3,Z,Z,var);
```

$$[[1 = \begin{bmatrix} 0 \end{bmatrix}],\ [1],\ \text{“Presentation”}]$$

The fourth integral cohomology $\mathrm{H}^4(S^1;\mathbb{Z}) = 0$:

```
> Ext(4,Z,Z,var);
```

$$[[1 = \begin{bmatrix} 0 \end{bmatrix}],\ [1],\ \text{“Presentation”}]$$

The fifth integral cohomology $\mathrm{H}^5(S^1;\mathbb{Z}) = 0$:

```
> Ext(5,Z,Z,var);
```

$$[[1 = \begin{bmatrix} 0 \end{bmatrix}],\ [1],\ \text{“Presentation”}]$$

etc...

```
> Ext(6,Z,Z,var);
```

$$[[1 = \begin{bmatrix} 0 \end{bmatrix}], [1], \text{“Presentation”}]$$

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References

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