3ExtensionCounterExample

Corollary 6.1 of [BB] states that for a 3-extension module to exist the YONEDA product of consecutive 1-cocycles must vanish. This worksheet gives the details to the example in Subsection 8.4 of [BB] where it is shown that this condition is not sufficient.

```
> restart;
with(homalg):with(Involutive):
‘homalg/default‘:=‘Involutive/homalg‘;
```

$$homalg/default := Involutive/homalg$$

Let $D = \mathbb{Q}[x, y]$:

```
> var:=[x,y];
```

$$var := [x,\ y]$$

We choose the modules as follows:

```
> M := [x,y];
```

$$M := [x,\ y]$$

```
> L := [x,y];
```

$$L := [x,\ y]$$

```
> K := [x^2*y];
```

$$K := [x^2\,y]$$

```
> N := [y];
```

$$N := [y]$$

We need 1-cocycles $\eta_L^M \in \mathrm{Ext}^1(M, L)$, $\eta_K^L \in \mathrm{Ext}^1(L, K)$ and $\eta_N^K \in \mathrm{Ext}^1(K, N)$:

```
> Ext(1,M,L,var);
```

$$[[[1,\ 0] = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ [0,\ 1] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}],\ [[0,\ y],\ [y,\ 0],\ [0,\ x],\ [x,\ 0]],\ \text{“Presentation”},\ 2,\ [0,\ 0]]$$

```
> etaML := matrix([[0], [1]]);
```

$$etaML := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

```
> Ext(1,L,K,var);
```

$$[[1 = \begin{bmatrix} x\,y \\ 0 \end{bmatrix}],\ [y,\ x],\ \text{“Presentation”},\ 1,\ [0,\ 0]]$$

```
> etaLK := matrix([[x*y], [0]]);
```

$$etaLK := \begin{bmatrix} x\,y \\ 0 \end{bmatrix}$$

```
> Ext(1,K,N,var);
```

$$[[1 = \begin{bmatrix} 1 \end{bmatrix}],\ [y],\ \text{“Presentation”},\ 1 + \frac{s}{1-s},\ [1,\ 0]]$$

```
> etaKN := [x];
```

$$etaKN := [x]$$

The YONEDA products of the consecutive 1-cocycles are both zero:

```
> YonedaProductOfCocycles(M,etaML,1,L,etaLK,1,K,var,
"return_abstract_generator");
```

$$0 = \begin{bmatrix} -x\,y \end{bmatrix}$$

```
> YonedaProductOfCocycles(L,etaLK,1,K,etaKN,1,N,var,
"return_abstract_generator");
```

$$0 = \begin{bmatrix} x \end{bmatrix}$$

The procedure `A3ExtensionModule` tries to solve the system of equations given in [BB]. In this example, no 3-extension module exists:

```
> A3ExtensionModule(M,etaML,L,etaLK,K,etaKN,N,var);
```

FAIL

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Date: 2008-02-27
Last modified: 2008-02-27 15:48

References

[BB] Mohamed Barakat and Barbara Bremer, *Higher Extension Modules and the Yoneda Product*, submitted http://wwwb.math.rwth-aachen.de/homalg). 1

[BCG+03] Y. A. Blinkov, C. F. Cid, V. P. Gerdt, W. Plesken, and D. Robertz, *The* MAPLE *Package* Janet*:* I. *Polynomial Systems.* II. *Linear Partial Differential Equations*, Proc. 6th Int. Workshop on Computer Algebra in Scientific Computing, Passau, Germany, 2003, (http://wwwb.math.rwth-aachen.de/Janet), pp. 31–40 and 41–54.

[BR] Mohamed Barakat and Daniel Robertz, `homalg` – *A meta-package for homological algebra*, accepted for publication in Journal of Algebra and its Applications. (arXiv:math.AC/0701146 and http://wwwb.math.rwth-aachen.de/homalg).

[BR08] ———, `homalg` *project*, 2003-2008, (http://wwwb.math.rwth-aachen.de/homalg).

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