2ExtensionModule_Z

This worksheet gives the details of the example in Subsection 8.1 of [BB].

```
> restart;
with(homalg): with(PIR):
‘homalg/default‘:=‘PIR/homalg‘;
```

$$homalg/default := PIR/homalg$$

Let $D = \mathbb{Z}$:

```
> var := [];
```

$$var := []$$

We choose $M = L = N = \mathbb{Z}/2\mathbb{Z}$:

```
> M := [2];
L := [2];
N := [2];
```

$$M := [2]$$

$$L := [2]$$

$$N := [2]$$

$\mathrm{Hom}(\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}/2\mathbb{Z}) \cong \mathrm{Ext}^1_{\mathbb{Z}}(\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}/2\mathbb{Z}) \cong \mathbb{Z}/2\mathbb{Z}$:

```
> Hom(M,L,var);
```

$$[[1 = [\ 1\ ]], [2], \text{“Presentation”}, [2], 0]$$

```
> Ext(1,M,L,var);
```

$$[[1 = [\ 1\ ]], [2], \text{“Presentation”}, [2], 0]$$

$Ext^2(M, N) = 0$, so the YONEDAproduct of any two 1-cocycles vanishes:

```
> Ext(2,M,N,var);
```

$$[[1 = [\ 0\ ]], [1], \text{“Presentation”}, [1], 0]$$

So for any choice of η_L^M and η_N^L $\mathrm{ExtMod}(\eta_L^M, \eta_N^L) \neq \emptyset$. For the cardinality of $\mathrm{ExtMod}(\eta_L^M, \eta_N^L)$ we compute the first extension group of cocycles:

```
> etaML := [1];
etaLN := [1];
```

$$etaML := [1]$$

$$etaLN := [1]$$

```
> Ext1Of2OneCocycles(M,etaML,L,etaLN,N,var);
```

$$[[1 = [\ 1\ ]], [1], \text{“Presentation”}, [1], 0]$$

```
> etaML := [1];
etaLN := [0];
```

$$etaML := [1]$$

$$etaLN := [0]$$

```
> Ext1Of2OneCocycles(M,etaML,L,etaLN,N,var);
```

$$[[1 = [\ 1\ ]], [1], \text{“Presentation”}, [1], 0]$$

```
> etaML := [0];
etaLN := [1];
```

$$etaML := [0]$$

$$etaLN := [1]$$

```
> Ext1Of2OneCocycles(M,etaML,L,etaLN,N,var);
```

$$[[1 = [\ 1\ ]], [1], \text{“Presentation”}, [1], 0]$$

```
> etaML := [0];
etaLN := [0];
```

$$etaML := [0]$$

$$etaLN := [0]$$

```
> Ext1Of2OneCocycles(M,etaML,L,etaLN,N,var);
```

$$[[1 = \left[\ 1\ \right]], [2], \text{“Presentation”}, [2], 0]$$

So

$$\mathrm{Ext}^1(\eta_L^M, \eta_N^L) = \begin{cases} \mathbb{Z}/2\mathbb{Z} & , \quad \text{if } \eta_L^M = (0) \text{ and } \eta_N^L = (0) \\ 0 & , \quad \text{else.} \end{cases}$$

and therefore

$$|\operatorname{ExtMod}(\eta_L^M, \eta_N^L)| = \begin{cases} 2 & , \quad \text{if } \eta_L^M = (0) \text{ and } \eta_N^L = (0) \\ 1 & , \quad \text{else,} \end{cases} .$$

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References

[BB] Mohamed Barakat and Barbara Bremer, *Higher Extension Modules and the Yoneda Product*, submitted (http://wwwb.math.rwth-aachen.de/homalg). 1

[BG08] Mohamed Barakat and Simon Görtzen, PIR*: A tiny* homalg *ring package for* Maple*-built-in principal ideal rings*, 2004-2008, (http://wwwb.math.rwth-aachen.de:8040).

[BR] Mohamed Barakat and Daniel Robertz, homalg – *A meta-package for homological algebra*, accepted for publication in Journal of Algebra and its Applications. (arXiv:math.AC/0701146 and http://wwwb.math.rwth-aachen.de/homalg).

[BR08] ———, homalg *project*, 2003-2008, (http://wwwb.math.rwth-aachen.de/homalg).

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