

Assignment sheet 2

PROF. DR. MOHAMED BARAKAT, M.SC. KAMAL SALEH

Exercise 1. (4 points)

Let $I \trianglelefteq R$ be an ideal. For $f, g \in R$ show that

$$\text{NF}(f + g, I) = \text{NF}(f, I) + \text{NF}(g, I), \text{ and}$$

$$\text{NF}(f \cdot g, I) = \text{NF}(\text{NF}(f, I) \cdot \text{NF}(g, I), I).$$

Exercise 2. (4 points)

Let $I = \langle m_1, \dots, m_r \rangle$ be a monomial ideal of R and let m be a monomial in R . Show that the so-called quotient ideal

$$I : m := \{r \in R \mid mr \in I\}$$

is generated by the monomials

$$\frac{\text{LCM}(m_i, m)}{m} = \frac{m_i}{\text{GCD}(m_i, m)}, \quad 1 \leq i \leq r.$$

In particular, $I : m$ is a monomial ideal as well.

Exercise 3. (4 points)

Let $f_1 = x^2y - y^3$, $f_2 = x^3 \in k[x, y]$ with $>_{\text{lex}}$. Compute a Gröbner basis for the ideal $I = \langle f_1, f_2 \rangle$. Visualize the monomials in $L(I)$, and compute a multiplication table for $k[x, y]/I$.

Hand in until May 16th 12:15 in the class or in Box in ENC, 2nd floor, at the entrance of the building part D.